HOMOLOGY: THEORETICAL AND COMPUTATIONAL ASPECTS

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Generalized persistent homologies:
G-invariant and multi-dimensional persistence

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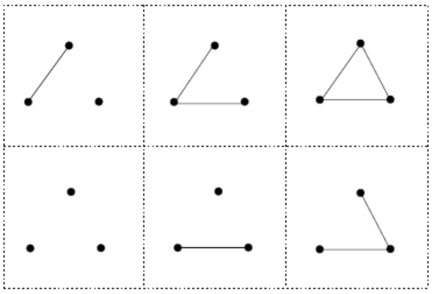
Exercises on Multidimensional Persistence

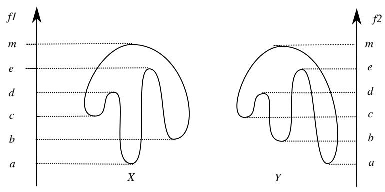
(1) Consider the following 2-dimensional filtration F of a circle:



- Construct the 2-dimensional persistence modules $\mathbf{M}_0 = H_0(F;k)$ and $\mathbf{M}_1 = H_1(F;k)$.
- Compute $\xi_0(\mathbf{M}_0)$, $\xi_1(\mathbf{M}_0)$, $\xi_2(\mathbf{M}_0)$, $\xi_0(\mathbf{M}_1)$

(2) Construct a CW-complex X together with a cellular function $f : X \to \mathbb{R}^2$ such that $H_1 \circ F(X, f; \mathbb{Q}) = \langle g_1, g_2 | r = g_1 - g_2 \rangle$ with $\mathrm{gr}(g_1) = (1,0)$, $\mathrm{gr}(g_2) = (0,1)$, $\mathrm{gr}(r) = (1,1)$.

(3) Consider the shapes (X, f), (X, f') given below:



Construct a 0-interleaving $\mathbf{h} : H_0 \circ F(X, f; \mathbb{Z}_2) \to H_0 \circ F(X', f'; \mathbb{Z}_2)$.

(4) Consider the 2-dimensional persistence modules over a field k given by:

$$\mathbf{M} = \{M_{(0,0)} = k^3, M_{(1,0)} = k, M_{(0,1)} = k^2, M_{(1,1)} = k, M_{(i,j)} = 0 \text{ otherwise}\}$$

$$\mathbf{N} = \{M_{(0,0)} = k^3, M_{(1,0)} = k^2, M_{(0,1)} = k^2, M_{(1,1)} = k, M_{(i,j)} = 0 \text{ otherwise}\}$$

(the maps are the obvious ones).
Compute the interleaving distance between $\mathbf{M}$ and $\mathbf{N}$.

(5) Given a 2-simplex $\sigma =< v_0, v_1, v_2 >$, compute the axis-wise linear interpolation $f^{\urcorner} : |\sigma| \to \mathbb{R}^2$ of the function $f : \{v_0, v_1, v_2\} \to \mathbb{R}^2$ defined on the vertices of σ as follows: $f(v_0) = (0,1)$, $f(v_1) = (1,0)$, $f(v_2) = (1,1)$.